Model Independent Results for SU(3) Violation in Light-Cone Distribution Functions

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Using chiral symmetry we investigate the leading SU(3) violation in light-cone distribution functions $\phi_M(x)$ of the pion, kaon, and eta. It is shown that terms non-analytic in the quark masses do not affect the shape, and only appear in the decay constants. Predictive power is retained including the leading analytic m_q operators. With the symmetry violating corrections we derive useful model independent relations between ϕ_π, ϕ_η, ϕ_{K^+,K^0}, and $\phi_{\bar{K}^0,K^-}$. Using the soft-collinear effective theory we discuss how factorization generates the subleading chiral coefficients.

At large momentum transfers $Q^2 \gg \Lambda_{\rm QCD}^2$, factorization in QCD dramatically simplifies hadronic processes. Observables depend on universal non-perturbative light cone distribution functions like $\phi_M(x,\mu)$ for exclusive processes with mesons M, or the parton distribution functions $f_{i/H}(x,\mu)$ for deep inelastic scattering on hadron H. A well known example is the electromagnetic form factor [1, 2], which dropping $\Lambda_{\rm QCD}/Q$ corrections is

$$F(Q^2) = \frac{f_a f_b}{Q^2}\int_0^1 dx\, dy\, T(x,y,Q,\mu)\phi_a(x,\mu)\phi_b(y,\mu)\,. \quad (1)$$

Here f_i are meson decay constants and the hard scattering kernel T is calculated perturbatively. For $a=b=\pi^\pm$, $T(x,y,\mu=Q) = 8\pi\alpha_s(Q)/(9xy) + \mathcal{O}(\alpha_s^2)$, which leads to inverse moments of the light-cone distribution functions. The same distributions ϕ_a, appear in many factorization theorems including those relevant to measuring fundamental parameters of the Standard Model [3], such as $B\to\pi\ell\nu, \eta\ell\nu$ which give the CKM matrix element $|V_{ub}|$, $B\to D\pi$ used for tagging, and $B\to\pi\pi, K\pi, K\bar{K}, \pi\eta, \ldots$ which are important for measuring CP violation.

QCD factorization theorems like the one in Eq. (1) make no attempt to separate the light quark mass scales $m_{u,d,s}$ from scales such as $\Lambda_{\rm QCD}$ or Λ_χ (the scale of chiral symmetry breaking). Thus $\phi_a(x,\mu)$ is a function of all of these scales. In the chiral limit $m_{u,d,s}\to 0$, SU(3) symmetry predicts a further relation $\phi_\pi = \phi_K = \phi_\eta = \phi_0$. For simplicity we work in the isospin limit and the $\overline{\rm MS}$ scheme, and normalize the distributions so that $\int dx\, \phi_M(x) = 1$. Generically from chiral symmetry the leading order SU(3) violation takes the form $[M=\pi,K,\eta]$

$$\phi_M(x,\mu) = \phi_0(x,\mu) + \sum_{P=\pi,K,\eta} \frac{m_P^2}{(4\pi f)^2}\Big[E_M^P(x,\mu)\ln\Big(\frac{m_P^2}{\mu_\chi^2}\Big) + F_M^P(x,\mu,\mu_\chi)\Big]\,. \quad (2)$$

The functions ϕ_0 , E_M^P, and F_M^P are independent of m_q, and are only functions of $\Lambda_{\rm QCD}$, μ, and x the momentum fraction of the quark in the meson at the point of the hard interaction. F_M^P also depends on the ChPT dimensional regularization parameter μ_χ which cancels the $\ln(m_P^2/\mu_\chi^2)$ dependence, so by construction ϕ_M is μ_χ independent. So far these sizeable $\sim 30\%$ corrections to the SU(3) limit have mostly been estimated in a model dependent fashion and even the signs of the effects remain uncertain. Recent light-cone sum rule results give the ratio of moments $\langle x_u^{-1}\rangle_{\pi^+}/\langle x_u^{-1}\rangle_{K^+} \simeq 1.12$ [4], correcting the earlier result $\langle x_u^{-1}\rangle_{\pi^+}/\langle x_u^{-1}\rangle_{K^+} \simeq 0.80$ [2, 5]. In $B\to MM'$ decays SU(3) violation was studied in [6].

In this letter we prove, by using chiral perturbation theory (ChPT), that at first order the light-cone distributions are analytic in m_q, meaning that

$$E_M^\pi(x) = 0\,, \qquad E_M^K(x) = 0\,, \qquad E_M^\eta(x) = 0\,. \quad (3)$$

Thus, chiral logarithms appear only in the decay constants f_M. We also derive relations between moments of the $F_M^P(x)$ coefficients, and thus determine model independent results for $\phi_M(x)$ that are valid at first order in the SU(3) violation, i.e. at the 10% level. Finally, we discuss quark mass effects using factorization and the soft-collinear effective theory [7] (SCET), and explain how ChPT and SCET results can be combined.

Recently ChPT has been applied to the computation of hadronic twist-2 matrix elements [8, 9]. Many applications have been worked out, e.g. chiral extrapolations of lattice data, generalized parton distributions [10], large N_C relations among distributions, and soft pion productions in deeply virtual Compton scattering [11]. It appears likely to us that parton SU(3) violation will be first quantitatively measured in meson light-cone distributions. This letter provides the necessary framework.

In momentum space the light-cone distribution functions can be defined by $\langle M^b|O^{A,a}(\omega_+,\omega_-)|0\rangle$, via

$$\langle M^b|(\bar\psi Y)_{\omega_1} \not{n}\gamma_5 \lambda^a (Y^\dagger\psi)_{\omega_2}|0\rangle = -i\delta^{ab}\delta\Big(\frac{\omega_- - n\cdot p_M}{2}\Big) \times f_M\, n\cdot p_M \int_0^1 dx\, \delta(\omega_+ - (2x-1)\, n\cdot p_M)\phi_M(x,\mu)\,, \quad (4)$$

where n is a light-like vector, $n^2=0$, $\omega_\pm = \omega_1 \pm \omega_2$, and our octet matrices are normalized so that ${\rm tr}[\lambda^a\lambda^b] = \delta^{ab}$. We use the subscript notation from SCET, so

that $(\bar{\psi}Y)_{\omega_1}$ is the Fourier transform of $\bar{\psi}(x^-)Y(x^-,\infty)$, where Y is a Wilson line of $n\cdot A$ gluons. Therefore ω_1 is the $n\cdot p$ momentum carried by this gauge invariant product of fields. Hard perturbative corrections will generate convolutions with coefficients $C(\omega_\pm, Q^2, \mu)$. We can expand $C(\omega_+, Q^2) = \sum_{k=0}^{\infty} C_k(-\omega_+)^k$ so that

$$\int d\omega_+\, C(\omega_+)\, O^{A,a}(\omega_+) = \sum_{k=0}^{\infty} C_k\, O_k^{A,a}\,, \tag{5}$$

where the tower of octet axial twist-2 operators are

$$O_k^{A,a} = \overline{\psi}\, \not{n}\gamma_5 \lambda^a \big[in\cdot \overleftrightarrow{D}\,\big]^k \psi\,. \tag{6}$$

Here $i\overleftrightarrow{D} = i\overleftarrow{D} - i\overrightarrow{D}$ and having the vector indices dotted into $n^{\mu_1}\cdots n^{\mu_{k+1}}$ has automatically projected onto the symmetric and traceless part. Comparing Eqs. (4) and (6) gives $[z=1-2x]$

$$\langle M^b|O_k^{A,a}|0\rangle = -if_M \delta^{ab}(n\cdot p_M)^{k+1}\big\langle z^k\big\rangle_M\,,$$
$$\big\langle z^k\big\rangle_M = \int_0^1 dx\,\big(1-2x\big)^k\,\phi_M(x)\,. \tag{7}$$

Thus, the matrix element of $O_k^{A,a}$ is related to moments of the meson light-cone distribution functions. A subscript $M=\eta$ denotes the purely octet part. It is convenient to work with $O_k^{A,a} = O_k^{R,a} - O_k^{L,a}$ where $O_k^{R,a} = \overline{\psi}_R \not{n}\lambda_R^a\big[in\cdot\overleftrightarrow{D}\,\big]^k\psi_R\,, O_k^{L,a} = \overline{\psi}_L\not{n}\lambda_L^a\big[in\cdot\overleftrightarrow{D}\,\big]^k\psi_L\,,$ and $\psi_{L,R} = (1\mp\gamma_5)/2\,\psi$. The distinction between λ_R^a, λ_L^a, and λ^a is for bookkeeping purposes, and we set $\lambda_{R,L}^a = \lambda^a$ at the end.

When $a=3$ or 8, $O_k^{A,a}$ transforms simply under charge conjugation ($\mathcal{C}$), being even when k is even, and odd when k is odd. The meson states π^0 and η (ie. $M^{3,8}$) are $\mathcal{C}$ even. Thus from Eq. (7), $\big\langle z^k\big\rangle_{\pi^0,\eta}$ vanish when k is odd due to $\mathcal{C}$ (and using isospin the same applies for $M=\pi^\pm$). For all a's the operator would transform as

$$\mathcal{C}^{-1}O_k^{A,a}\mathcal{C} = (-1)^k O_k^{A,a}\,, \tag{8}$$

if we demanded that under the $\mathcal{C}$ transformation

$$\lambda_R^a \to \lambda_L^{a\,T}\,,\qquad \lambda_L^a\to\lambda_R^{a\,T}\,. \tag{9}$$

Eq. (9) will be used to reproduce the $\mathcal{C}$ violating properties of $O_k^{A,a}$ when matching to the hadronic operators.

To construct the hadronic ChPT operators we define $\Sigma = \exp(2i\pi^a\lambda^a/f)$ and $m_q = \mathrm{diag}(\overline{m},\overline{m},m_s) = m_q^\dagger$. Under a chiral $SU(3)_L\times SU(3)_R$ transformation

$$\Sigma \to L\Sigma R^\dagger,\qquad m_q\to L m_q R^\dagger\,,$$
$$\lambda_R^a \to R\lambda_R^a R^\dagger,\qquad \lambda_L^a\to L\lambda_L^a L^\dagger. \tag{10}$$

Under charge conjugation $\Sigma\to\Sigma^T$, while $\lambda_{R,L}^a$ transform according to Eq. (9). At next to leading order (NLO) in the p^2/Λ_χ^2 and m_M^2/Λ_χ^2 chiral expansion

$$O_k^{A,a} \longrightarrow \sum_i c_{k,i}\mathcal{O}_{k,i}^{A,a} + \sum_i b_{k,i}\overline{\mathcal{O}}_{k,i}^{A,a}\,, \tag{11}$$

FIG. 1: NLO loop diagrams, where here $\otimes$ denotes an insertion of $\mathcal{O}_{k,0}^{A,a}$, and the dashed lines are meson fields.

where the $\mathcal{O}$'s are leading order (LO) and the $\overline{\mathcal{O}}$'s are NLO. The sum on i runs over hadronic operators having the same transformation properties as $O_k^{A,a}$. The ChPT Wilson coefficients $c_{k,i}$ and $b_{k,i}$ encode physics at the scale $p^2\sim\Lambda_\chi^2$ and the operators encode $p^2\ll\Lambda_\chi^2$.

At LO in the chiral expansion only one operator contributes in our analysis

$$\mathcal{O}_{j-1,0}^{A,a} = \frac{f^2}{8}\,\mathrm{Tr}\Big[\lambda_R^a\Big\{\Sigma^\dagger\Box^j\Sigma + (-1)^j(\Box^j\Sigma^\dagger)\Sigma\Big\}$$
$$-\lambda_L^a\Big\{\Sigma\Box^j\Sigma^\dagger + (-1)^j(\Box^j\Sigma)\Sigma^\dagger\Big\}\Big]\,, \tag{12}$$

where $\Box^j = (in\cdot\partial)^j$ and the factors of f have been inserted by using chiral counting rules ($f=f_M$ in the chiral limit). All other $\mathcal{O}_{k,i}^{A,a}$ operators have $\Box^{k+1}$ factors acting on more than one Σ field and as we will see, do not contribute to the $0\to M^b$ matrix element up to NLO.

Under charge conjugation, $\mathcal{O}_{k,0}^{A,a}\to(-1)^k\,\mathcal{O}_{k,0}^{A,a}$. Thus, when k is odd this operator gives vanishing $0\to M^b$ matrix elements, as expected by $\mathcal{C}$ and SU(3) symmetry. For k even

$$\langle M^b|c_{k,0}\mathcal{O}_{k,0}^{A,a}|0\rangle = -if_M\delta^{ab}(n\cdot p_M)^{k+1}c_{k,0}\,, \tag{13}$$

and comparing with Eq. (7) we see that

$$c_{k,0} = \langle z^k\rangle_0 = \int_0^1 dx\,(1-2x)^k\phi_0(x)\,, \tag{14}$$

where ϕ_0 is the distribution function in the SU(3) limit. Note that $c_{0,0}=1$ due to our normalization for ϕ_M.

At NLO chiral logarithms can be obtained from loop diagrams involving the LO operators as shown in Fig. 1. For $k=0$ the operator $O_{k=0}^{A,a}$ is the axial current, while $\mathcal{O}_{k=0,0}^{A,a}$ is just the standard ChPT axial current operator whose Fig. 1 graphs give the one-loop corrections to f_M. For odd k the one-loop graphs vanish since adding the chiral Lagrangian does not change the $\mathcal{C}$-invariance argument. For any $k>0$ the diagrams have a term where all derivatives act on the outgoing meson line, and this gives the same corrections as for f_M. The first diagram could have additional contributions from derivatives acting inside the loop but it is straightforward to show that these diagrams vanish identically since $n^2=0$, and that the same holds true for LO operators with derivatives on more than one Σ [8]. Thus, we have shown that all possible non-analytic corrections are contained in f_M at NLO. This is true for every moment, and so we conclude that the leading order SU(3) violation of $\phi_M(x)$ is analytic in m_q.

Analytic corrections are also generated by subleading operators, and at NLO we find the basis $[B_0 = -2\langle\bar\psi\psi\rangle/f^2]$

$$\begin{aligned}\overline{\mathcal{O}}^{A,a}_{j-1,1} &= 2B_0 \,\mathrm{Tr}\big[m_q\Sigma^\dagger + \Sigma m_q^\dagger\big]\;\mathrm{Tr}\Big[\lambda^a_R\Big\{\Sigma^\dagger\Box^j\Sigma + (-1)^j(\Box^j\Sigma^\dagger)\Sigma\Big\} - \lambda^a_L\Big\{\Sigma\Box^j\Sigma^\dagger + (-1)^j(\Box^j\Sigma)\Sigma^\dagger\Big\}\Big]\,,\\ \overline{\mathcal{O}}^{A,a}_{j-1,2} &= 2B_0\,\mathrm{Tr}\Big[\lambda^a_R\big\{m_q^\dagger\Box^j\Sigma + (-1)^j\Box^j\Sigma^\dagger m_q\big\} - \lambda^a_L\big\{m_q\Box^j\Sigma^\dagger + (-1)^j\Box^j\Sigma m_q^\dagger\big\}\Big]\,. \end{aligned} \quad (15)$$

All other NLO operators have derivatives on more than one Σ, or can be reduced to $\overline{\mathcal{O}}^{A,a}_{j-1,1}$ and $\overline{\mathcal{O}}^{A,a}_{j-1,2}$ using the equations of motion. For instance, consider

$$\begin{aligned}\overline{\mathcal{O}}^{A,a}_{j-1,3} = 2B_0\mathrm{Tr}\Big[&\lambda^a_R\big\{\Sigma^\dagger m_q\Sigma^\dagger\Box^j\Sigma + (-1)^j(\Box^j\Sigma^\dagger)\Sigma m_q^\dagger\Sigma\big\}\\ &-\lambda^a_L\big\{\Sigma m_q^\dagger\Sigma\Box^j\Sigma^\dagger + (-1)^j(\Box^j\Sigma)\Sigma^\dagger m_q\Sigma^\dagger\big\}\Big]\,. \end{aligned} \quad (16)$$

The sum and difference $\overline{\mathcal{O}}^{A,a}_{k,2} \pm \overline{\mathcal{O}}^{A,a}_{k,3}$ contain factors of $(\Sigma^\dagger m_q \pm m_q^\dagger\Sigma)$ and $(\Sigma m_q^\dagger \pm m_q\Sigma^\dagger)$. Using the equations of motion for Σ

$$\Sigma^\dagger(i\partial_\mu)^2\Sigma = -(i\partial^\mu\Sigma^\dagger)(i\partial_\mu\Sigma) + B_0(\Sigma^\dagger m_q - m_q^\dagger\Sigma)\,, \quad (17)$$

together with the analogous equation for $\Sigma^\dagger$ we can trade $\overline{\mathcal{O}}^{A,a}_{k,2} - \overline{\mathcal{O}}^{A,a}_{k,3}$ for operators with derivatives on more than one Σ. These additional operators do not generate one-meson matrix elements at tree level and can be omitted from our analysis. Thus only $\overline{\mathcal{O}}^{A,a}_{k,2} + \overline{\mathcal{O}}^{A,a}_{k,3}$ contributes and for simplicity we trade this for $\overline{\mathcal{O}}^{A,a}_{k,2}$. We can also consider operators analogous to $\overline{\mathcal{O}}^{A,a}_{k,3}$ but with the $\Box^k$ factors on a different Σ. Since $\Box^k(\Sigma^\dagger\Sigma) = 0$ we can use

$$0 = (\Box^k\Sigma^\dagger)\Sigma + \Sigma^\dagger(\Box^k\Sigma) + \ldots\,, \quad (18)$$

where the ellipse denote $(\Box^{k-m}\Sigma^\dagger)(\Box^m\Sigma)$ terms that only contribute for matrix elements with more than one meson. Thus, Eq. (18) allows us to move factors of $\Box^k$ onto a neighboring Σ and eliminate operators. Finally, we can consider operators where the power suppression is generated by derivatives rather than a factor of m_q. Boost invariance requires that these operators still have j factors of n^μ, so they will involve $\Box^j$ just like the operators we have been considering. To get power suppression with derivatives we can either use $(\partial_\mu\Sigma^\dagger)(\partial^\mu\Sigma)$ which has derivatives on more than one Σ, or $\Sigma^\dagger(\partial^\mu)^2\Sigma$ which can be traded for operators with m_q's using Eq. (17). Therefore, the operators with m_q's in Eq. (15) suffice.

At NLO we need $\langle\overline{\mathcal{O}}^{A,a}_{k,i}\rangle$ at tree level. The $k = 0$ operators are equivalent to those derived from the standard $\mathcal{O}(p^4)$ chiral Lagrangian, so $b_{0,1} = L_4$ and $b_{0,2} = L_5$. Adding the wavefunction counterterms we find $\langle\pi^b|c_{k,0}Z^{1/2}\mathcal{O}^{A,a}_{k,0} + \sum_i b_{k,i}\overline{\mathcal{O}}^{A,a}_{k,i}|0\rangle = N_k A_k$, where $N_k = -if(n\cdot p_M)^{k+1}$ and the NLO contribution is

$$\begin{aligned}A_k = \frac{4B_0}{f^2}\Big\{&\big[1-(-1)^{k+1}\big]2\mathrm{Tr}[m_q]\delta^{ab}(2b_{k,1} - L_4c_{k,0})\\ &+\mathrm{Tr}\big[m_q\{\lambda^a\lambda^b - (-1)^{k+1}\lambda^b\lambda^a\}\big](2b_{k,2} - L_5c_{k,0})\Big\}\,. \end{aligned} \quad (19)$$

For $k = 0$, A_0 gives the standard counterterm corrections to f_M at NLO, which are combined with the one-loop contributions from Fig. 1. Numerically

$$\frac{f_K}{f_\pi} = 1.23\,, \qquad \frac{f_\eta}{f_\pi} = 1.33\,. \quad (20)$$

To compute the analytic chiral corrections to the moments of $\phi_M(x)$ we subtract the corrections to f_M,

$$\begin{aligned}\Delta A_k = \frac{8B_0}{f^2}\Big\{&\big[1-(-1)^{k+1}\big]2\mathrm{Tr}[m_q]\delta^{ab}(b_{k,1} - L_4c_{k,0})\\ &+\mathrm{Tr}\big[m_q\{\lambda^a\lambda^b - (-1)^{k+1}\lambda^b\lambda^a\}\big](b_{k,2} - L_5c_{k,0})\Big\}\,. \end{aligned} \quad (21)$$

For odd k (odd moments), ΔA_k collapses to the $\mathrm{Tr}\big[m_q\big[\lambda^a, \lambda^b\big]\big]$. This yields $[m = 0, 1, 2, \cdots]$

$$\begin{aligned}\big\langle z^{2m+1}\big\rangle_\pi &= \big\langle z^{2m+1}\big\rangle_\eta = 0\,, \quad (22)\\ \big\langle z^{2m+1}\big\rangle_{K^0} &= \big\langle z^{2m+1}\big\rangle_{K^+} = -\big\langle z^{2m+1}\big\rangle_{\overline{K}^0} = -\big\langle z^{2m+1}\big\rangle_{K^-}\\ &= \frac{8B_0(m_s - \bar m)}{f^2}\; b_{2m+1,2}\,. \end{aligned}$$

The leading SU(3) violation for odd k agrees with our expectations based on $\mathcal{C}$ and isospin symmetry. For even k (even moments), the ΔA_k gives structures $\delta^{ab}\mathrm{Tr}[m_q]$ and $\mathrm{Tr}[m_q\{\lambda^a, \lambda^b\}]$ so

$$\begin{aligned}\delta\big\langle z^{2m}\big\rangle_\pi &= 2\overline{m}\,\alpha_{2m} + (2\overline{m} + m_s)\beta_{2m}\,, \quad (23)\\ \delta\big\langle z^{2m}\big\rangle_K &= (\overline{m} + m_s)\alpha_{2m} + (2\overline{m} + m_s)\beta_{2m}\,,\\ \delta\big\langle z^{2m}\big\rangle_\eta &= \frac{(2\overline{m} + 4m_s)}{3}\,\alpha_{2m} + (2\overline{m} + m_s)\beta_{2m}\,, \end{aligned}$$

where δ means the deviation from the chiral limit value, $\alpha_{2m} = 8B_0(b_{2m,2} - L_5c_{2m,0})/f^2$ and $\beta_{2m} = 32B_0(b_{2m,1} - L_4c_{2m,0})/f^2$. By isospin and charge conjugation the even moments of different pion states (or kaon states) are equal. Eq. (23) implies a Gell-Mann-Okubo-like relation

$$\big\langle z^{2m}\big\rangle_\pi + 3\big\langle z^{2m}\big\rangle_\eta = 4\big\langle z^{2m}\big\rangle_K\,. \quad (24)$$

The moment relations in Eqs. (22,24) imply relations among the meson light cone wave functions, namely

$$\begin{aligned}\phi_\pi(x,\mu) + 3\phi_\eta(x,\mu) &= 2\big[\phi_{K^+}(x,\mu) + \phi_{K^-}(x,\mu)\big]\\ &= 2\big[\phi_{K^0}(x,\mu) + \phi_{\overline{K}^0}(x,\mu)\big]\,, \end{aligned} \quad (25)$$

which is valid including the leading SU(3) violation. They also imply useful relations among the frequently used Gegenbauer moments, defined by

$$a_n^M(\mu) = \frac{4n+6}{6+9n+3n^2}\int_0^1\! dx\, C_n^{3/2}(2x-1)\phi_M(x,\mu)\,, \quad (26)$$

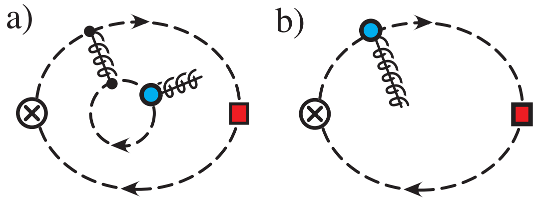

FIG. 2: Quark mass contributions to $\phi_M(x)$ factorization. Here the $\otimes$ is an insertion of $O^{A,a}$, the shaded circle is $\mathcal{L}_m^{(0)}$, and the box is the meson interpolating field. Here dashed lines are collinear quarks and springs are collinear gluons.

with $a_0 = 1$. Here $C_n^{3/2}(z)$ denote the Gegenbauer polynomials which are even (odd) functions of z when n is even (odd). Eqs. (22) and (24) imply that

$$\begin{aligned} 4a_{2m}^K &= a_{2m}^\pi + 3a_{2m}^\eta \,, \qquad a_{2m+1}^\pi = a_{2m+1}^\eta = 0\,, \\ a_{2m+1}^{K^0} &= a_{2m+1}^{K^+} = -a_{2m+1}^{\overline{K^0}} = -a_{2m+1}^{K^-}\,. \end{aligned} \tag{27}$$

In QCD or SCET factorization theorems it is often the inverse moments with respect to the quark or antiquark that appear, $\langle x_{q,\bar q}^{-1}\rangle_M = 3[1+\sum_{n=1}^\infty (\pm 1)^n a_n^M]$. We find

$$\begin{aligned} &\big\langle x_q^{-1}\big\rangle_\pi + 3\big\langle x_q^{-1}\big\rangle_\eta = 2\big[\big\langle x_q^{-1}\big\rangle_{K^+} + \big\langle x_q^{-1}\big\rangle_{K^-}\big], \\ &\big\langle x_s^{-1}\big\rangle_{K^-} - \big\langle x_{\bar u}^{-1}\big\rangle_{K^-} = \big\langle x_{\bar s}^{-1}\big\rangle_{K^+} - \big\langle x_u^{-1}\big\rangle_{K^+}\,, \end{aligned} \tag{28}$$

with identical expressions for the antiquark $\langle x_{\bar q}^{-1}\rangle$ in line 1 and for $\{K^-, K^+\} \to \{\bar K^0, K^0\}$ with $u \to d$ in line 2.

Finally, we relate the m_q ChPT corrections to m_q's in quark level factorization in SCET. Hard-collinear factorization is simplest to derive in a Breit frame where the quarks are collinear, with fields ξ_n. After a field redefinition their LO action is [7]

$$\mathcal{L}_{\xi\xi}^{(0)} = \bar\xi_n\Big[in\cdot D_c + (i\not{D}_\perp^c - m_q)\frac{1}{i\bar n\cdot D_c}(i\not{D}_\perp^c + m_q)\Big]\frac{\not{\bar n}}{2}\xi_n\,,$$

with the dependence on the matrix m_q from Ref. [12]. The linear m_q term is chiral odd and can be written

$$\mathcal{L}_m^{(0)} = (\bar\xi_n W) m_q \Big[\frac{1}{\bar{\mathcal{P}}} ig\not{\mathcal{B}}_\perp\Big]\frac{\not{\bar n}}{2}(W\xi_n)\,. \tag{29}$$

Here $ig\not{\mathcal{B}}_\perp = 1/\bar{\mathcal{P}}W^\dagger[i\bar n\cdot D_c, i\not{D}_{c,\perp}]W$ and W is a Wilson line of collinear $\bar n\cdot A_n$ fields. Eq. (29) makes it clear that the Feynman rules have ≥ 1 collinear gluon. Similarly the chiral condensate from [12] can be written

$$\big\langle\Omega\big|\,(\bar\xi_{n,R}^{(i)} W)\big[\frac{1}{\bar{\mathcal{P}}} ig\not{\mathcal{B}}_\perp\big]\frac{\not{\bar n}}{2}(W^\dagger\xi_{n,L}^{(j)})\big|\Omega\big\rangle = v\,\delta^{ij}\,. \tag{30}$$

$\mathcal{L}_m^{(0)}$ is suppressed relative to the $m_q = 0$ terms in $\mathcal{L}_{\xi\xi}^{(0)}$ by $m_q/\Lambda_{\rm QCD}$ and gives the complete set of these corrections. Thus these corrections are universal, they depend only on the distribution function and *not* on the underlying hard process which led to $O^{A,a}$ in the first place. (If we also want m_q/Q corrections, then power corrections to $O^{A,a}$ also contribute.) Thus, at $\mathcal{O}(m_q/\Lambda_{\rm QCD})$ we can simply use states in the chiral limit and add the time-ordered product

$$\int d^4y\; \mathrm{T}\big[O^{A,a}(\omega_\pm)(0)\, i\mathcal{L}_m^{(0)}(y)\big] = T_m^{(S)} + T_m^{(V)}\,. \tag{31}$$

The quark fields in $\mathcal{L}_m^{(0)}$ can either contract with themselves to give $T_m^{(S)}$ as in Fig. 2a ("sea" contribution), or contract with a quark field in $O^{A,a}$ to give $T_m^{(V)}$ as in Fig. 2b ("valence" contribution). Naively the Dirac structure in Eq. (31) cause the matrix elements to vanish as they are odd in $\not{D}_\perp$, however the condensate in Eq. (30) makes them non-zero. This may provide a generic mechanism for inferring the presence of "chirally enhanced" condensate terms. It also agrees with Eqs. (22,23) where $\delta\langle x^k\rangle_M \propto \langle\bar\psi\psi\rangle$. Using Eq. (31) and removing f_M, we can work out an explicit factorization theorem where $T_m^{(S)}$ ($T_m^{(V)}$) gives moments related to the chiral coefficient $b_{k,1}$ ($b_{k,2}$). This is beyond the scope of this letter.

An obvious place to study SU(3) violation in ϕ_M are π, K, and η form factors, but also decay processes including $\bar B^0 \to D_s K^-$ [13], $B \to M$ radiative decays, and $B \to MM'$. Our results show that only linear m_q dependence is required for lattice QCD extrapolations of ratios of ϕ_M moments. Measuring a difference between the s and u inverse moments for kaons in Eq. (28) provides a simple way of observing first order SU(3) violation. The relation in Eq. (24) could be used to disentangle η-η' mixing.

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